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IMPROVING THE ANALYTICAL REPRESENTATION OF THE GEOMAGNETIC FIELD ACCORDING TO SPUTNIKS' DATA

In order to solve a series of geophysical problems it is imperative to possess a sufficiently reliable analytical understanding of the geomagnetic field. The spherical harmonic progression with coefficients modified in time, is one of the most expedient manner of presenting the geomagnetic field. There should be a possibility of analyzing, in connection with this type of action, the data obtained during a short period of time and which should not be reduced to one period. Such data are obtained from the measurements taken by the Sputniks of the geomagnetic field, meaning measurements in directly non-uniformly distributed and taken in the course of a comparatively short period of time.

Yet, as a rule, such measurements produce only the full intensity modulus of the geomagnetic field T, with X, Y and Z components (the north, east and vertical components, respectively) which can be represented by spherical harmonic progressions, linear in relation to the coefficients g_n^m and h_n^m .

$$X=\sum_{n=1}^{\infty}\sum_{m=0}^{n}(g_n^m\cos m\lambda+h_n^m\sin m\lambda)\,\frac{dP_n^m(\cos\theta)}{d\theta}\cdot\left(\frac{R}{R+h}\right)^{n+2},$$

$$Y=\sum_{n=1}^{\infty}\sum_{m=1}^{n}(g_n^m\sin m\lambda-h_n^m\cos m\lambda)\frac{m}{\sin\theta}\cdot P_n^m(\cos\theta)\left(\frac{R}{R+h}\right)^{n+2}, \qquad (1)$$

$$Z=-\sum_{n=1}^{\infty}\sum_{m=0}^{n}(n+1)(g_n^m\cos m\lambda+h_n^m\sin m\lambda)P_n^m(\cos\theta)\left(\frac{R}{R+h}\right)^{n+2},$$

$$T=\left[X^2+Y^2+Z^2\right]^{1/2},$$

where $P_n^m(\cos\Theta)$ are the added Legendre polynomials of the first type, Θ - supplemental width, λ - length, R - the Earth radius, h - altitude above the Earth surface, g_n^m and h_n^m coefficients. The analytical presentation by means of formula (1) offers no possibility of dividing the fields into the internal and external parts.

When the components measurements are on hand the analysis is reduced to the solution of the redefined system of linear equations. As a rule, the method of the least squares is used.

The modulus of the full field intensity is a quadratic function in relation to coefficients. In such a case the redefined system of non-linear equations should be solved. The procedure to be used in a direct solution of such system is described in article (1) for two harmonics only. As the number of harmonics increases, this method becomes too cumbersome and inapplicable, for all practical purposes.

The approximate interation method is the most acceptable one (2). In this case definition affects the corrections to the coefficients the Δg_n^m and Δh_n^m. instead of the coefficients g_n^m, and h_n^m, . Let us assume that

$$T(G)=\left\{[X(G)]^2+[Y(G)]^2+[Z(G)]^2\right\}^{1/2}, \qquad (2)$$

where X(G), Y(G) and Z(G) ~~is~~ are a short recording of spherical harmonic progressions which can represent respectively, the north, east and vertical components of the geomagnetic field.

We understand that G is the cumulative coefficients g_n^m and h_n^m. . If G_0 is a certain nul value for which a combination of coefficients of any analysis can be taken, the sought for value G will be $G_0 + \Delta G$ (ΔG-short recording Δg_n^m and Δh_n^m) and (2) will then appear as:

$$T(G_0+\Delta G)=\left\{[X(G_0+\Delta G)]^2+[Y(G_0+\Delta G)]^2+[Z(G_0+\Delta G)]^2\right\}^{1/2}. \quad (3) \qquad (3)$$

By resolving $T(G_0 + \Delta G)$ into Taylor series and confining ourselves to the first member we obtain:

$$T(G_0+\Delta G)\cong T(G_0)+\sum_{j=1}^{n(n+2)}\left[\frac{\partial T(G)}{\partial g_j}\right]_{G=G_0}\cdot\Delta g_j \, , \quad (4) \qquad (4)$$

where $\sum_{j=1}^{n(n+2)} \left[\frac{\partial T(G)}{\partial g_j}\right]_{G=G_0}$ is the sum of quotients derivatives from $T(G)$ of each coefficient G_j (j being the ordinal number of a series of coefficients $g^{\circ}{}_1, g^1{}_1, g^{\circ}{}_2, \ldots, g_n^m, h^1{}_1, h^1{}_2, \ldots, h_n^m$) at $G = G_0$;

n is the number of harmonics.

In this way, each measured value ($T_{meas.}$) may be presented approximately in this form:

$$(T_{u3M})_i \cong T_i(G_0) + \sum_{j=1}^{n(n+2)} \left[\frac{\partial T_i(G)}{\partial g_j}\right]_{G=G_0} \cdot \Delta g_j \quad \text{or}$$

$$\Delta T_i \cong \sum_{j=1}^{n(n+2)} a_{ij} \cdot \Delta g_j , \qquad \text{where} \quad (s)$$

$$\Delta T_i = (T_{u3M})_i - T_i(G_0) \quad u$$

$$a_{ij} = \frac{X_i(G_0)}{T_i(G_0)} \cos m\lambda_i \left[\frac{dP_n^m(\cos\theta)}{d\theta}\right]_{\theta=\theta_i} +$$
$$+ \left[\frac{Y_i(G_0)}{T_i(G_0)} \cdot \frac{m}{\sin\theta_i} \sin m\lambda_i - (n+1)\frac{Z_i(G_0)}{T_i(G_0)} \cdot \cos m\lambda_i\right] \cdot P_n^m(\cos\theta_i)$$

at Δg_n^m and

$$a_{ij} = \frac{X_i(G_0)}{T_i(G_0)} \cdot \sin m\lambda_i \left[\frac{dP_n^m(\cos\theta)}{d\theta}\right]_{\theta=\theta_i} -$$
$$- \left[\frac{Y_i(G_0)}{T_i(G_0)} \cdot \frac{m}{\sin\theta_i} \cdot \cos m\lambda_i + (n+1)\frac{Z_i(G_0)}{T_i(G_0)} \sin m\lambda_i\right] \times$$
$$\times P_n^m(\cos\theta_i) \quad npu \; \Delta h_n^m ,$$

in other words, in each point measured the difference between the measured and the calculated - using the nul coefficients values (ΔT_i) is presented by a linear function in relation to corrections in the nul coefficients (ΔG).

Fig. 1

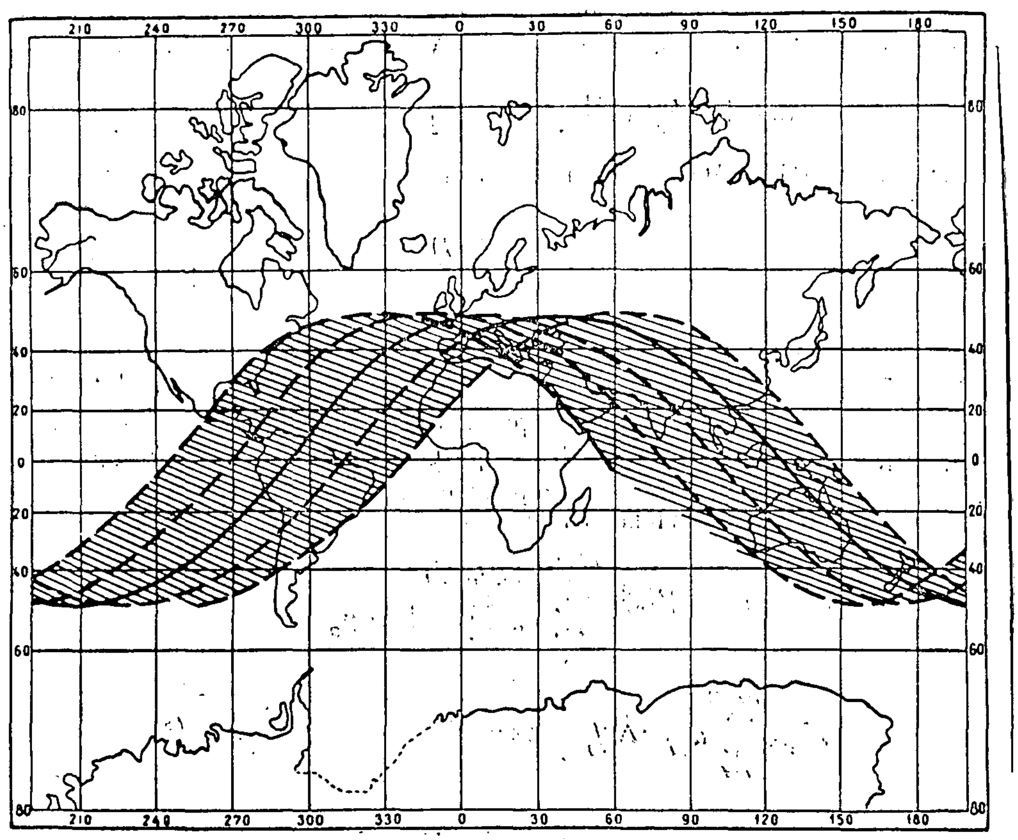
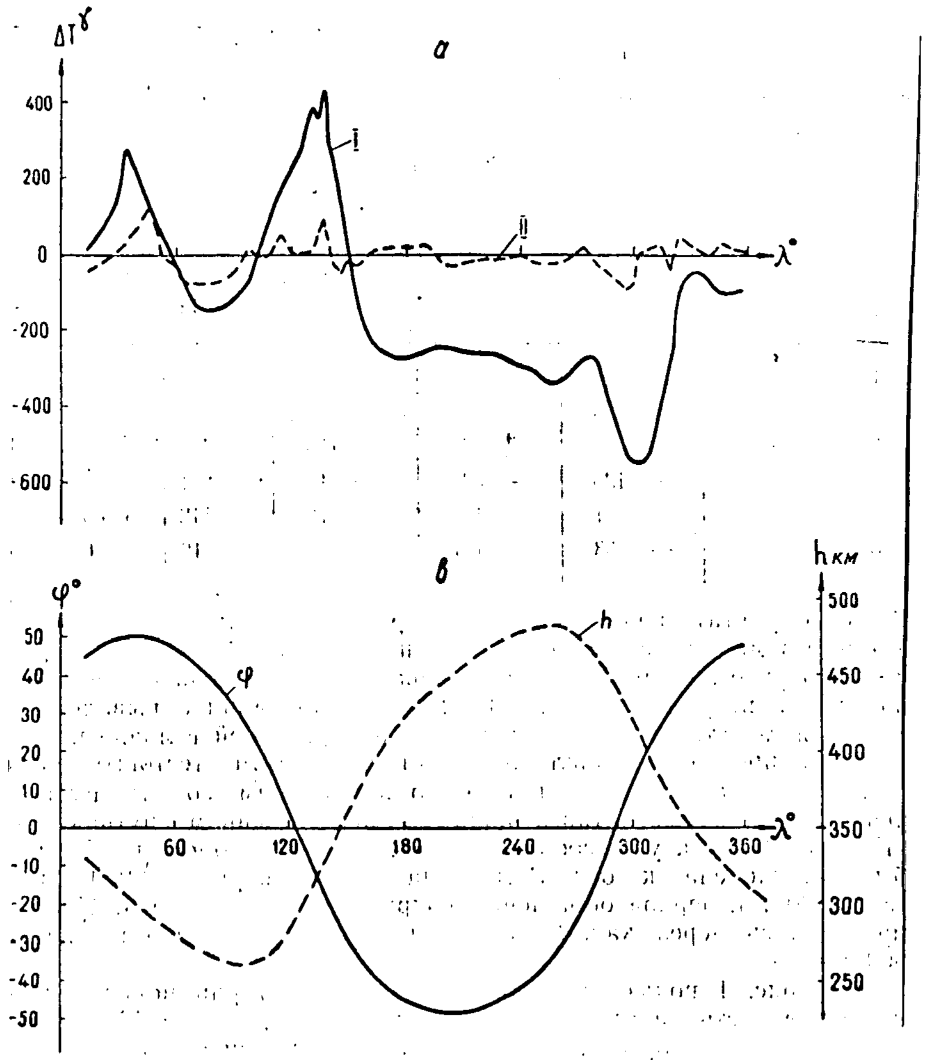


Fig. 2



The solution of system (5) is possible only with the aid of computers. A preliminary program made up for the M-20 computer enables to determine ΔG) corrections in the first 48 coefficients (n = 6) of the spherical progression. Coordinates of all points (h - altitude, φ -width, λ-length) and the measured field values (T_{meas}) were recorded on a magnetic drum; the ΔT_i value for each point was calculated and where it did not exceed (in relation to the modulus) a certain pre-established value, a (5) equation was built for such point, and, later on, a conditional equation was deduced.

The redefinition of the equation system was eliminated by using the method of the least squares. The entire matrix was composed based on one equation; this matrix was preserved in the work memory. All other equations were accounted for in a line-by-line summation of the matrix elements. A standard program (SI-103) (3) was used to solve the linear equations systems.

	0	1	2	3	4	5	6
1	—30415	— 2117					
2	— 1433	+ 2952	+ 1650				
3	+ 1242	— 1952	+ 1287	+ 841			
4	+ 979	+ 798	+ 495	— 363	+ 286		
5	— 210	+ 360	+ 226	— 36	— 173	— 62	
6	+ 70	+ 19	— 1	— 263	+ 12	— 7	— 90
1		+ 5820					
2		— 1998	+ 232				
3		— 433	+ 206	— 170			
4		+ 136	— 304	— 14	— 221		
5		+ 11	+ 118	— 76	— 112	+ 84	
6		— 33	+ 117	+ 41	— 16	— 6	— 1

As a result of solutions - the equation systems obtained the ΔG values which were summed up in the computer with the zero approximation G_o ; the first approximation obtained was used - without removing it from the computer - to obtain the second, etc. The iteration procedure would stop when the ΔG values became less than the pre-established value.

To check on the correctness of the methodology, after the test calculations, the first data on measurements of the modulus of the full intensity of the geomagnetic field on the earth satellite "Cosmos-49" were computed. As has already been reported (4) the orbital plane of "Cosmos-49" was slanted about 49° to the Earth axis, the perigee-about 260 km, the apogee about 490 km, the time of one orbit around the Earth was about 1.5 hours. The measurements were made each 32.8 sec. during the flight,in other words - about every 250 to 300 km.

Fig. 1 shows a section of the projection of the trajectory of the satellite flight upon the Earth surface. 420 measurements were made on this section (amounting to about 5 turns) which were then subjected to the processes described above.

The ΔT points counted did not exceed 1000 γ . Seven iterations were performed and the corrections ΔG of the last iteration did not exceed several tenths of gammas.

Fig. 2 (a and b) shows the results of analysis of one of those turns. Fig. 2 (b) offers an understanding of the coordinates (h, φ, λ) of the points. Fig. 2(a) shows, in curve 1 the difference between the measured field (T_{meas}) and that computed by means of coefficients of zero approximation $T(G_o)$. The coefficients of the analysis of the world magnetic maps of 1960 (5) were accepted for the G_o zero approximation (the median received from analyses along X, Y and Z). These coefficients were adjusted to SV (6) and corrected considering the ellipticity of the Earth (7). The numerical value of the zero approximation coefficients is presented in Table 1.

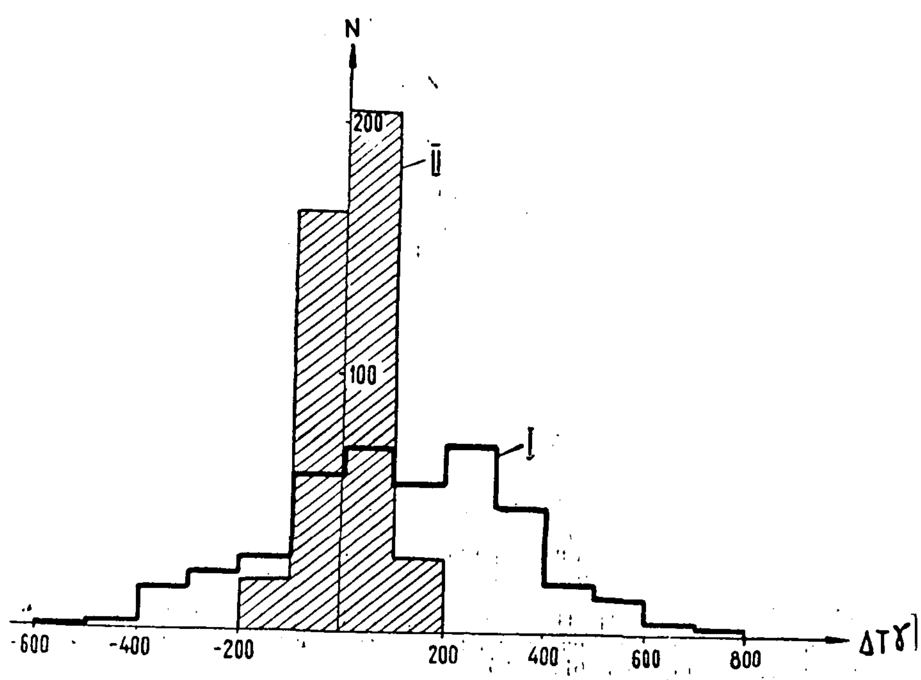
The magnitude and the character of the difference between the measured and the computed new coefficients values of the field are shown in Fig. 2(a) by curve II. The value of the coefficients obtained after 7 iterations are given in Table 2.

A comparison of curves I and II in Fig. 2 reveals that in the result of the analysis the magnitude of the difference between the measured and the computed values has not only been considerably decreased, but that its spectral content has been changed. If in a curve I the difference on the order of 400 to about 500 gamma have a considerable length (up to 90°to 100° longitude) which proves the insufficient representation of low harmonics, the curve II has differences which do not exceed

200 gamma in the first place, and secondly , their length does not exceed 20° to about 25° along the longitude.

Fig. 3 presents histograms which make possible to consider the improved analytical understanding of the geomagnetic field, in other words, achieving greater precision of coefficients for all 420 measured points. The ΔT differences are marked on the horizontal, the vertical N shows the number of points possessing these differences. Histogram I has the zero approximation coefficients, while histogram II carries coefficients after 7 iterations.

Fig. 3



As seen in Figures 2 and 3, coefficients obtained from the analysis made of a limited number of points for 6 harmonics present an improved analytical representation of the measured field. However, the coefficients shown in Table 2 present a good agreement with the measured values only over those spots of the Earth surface where these measurements had been taken (the hatched section of map in Fig. 1; the projection of a part of the trajectory shown in Fig.2 is represented with a solid line).

The coefficients obtained for the Earth surface beyond the hatched section limits may appear to be insufficiently representative not only due to the limited territory over which the measurements were performed, but also due to the limitations of the spherical progression (only 6 harmonics) and number of points subjected to the analysis.

The result obtained on the basis of analysis of the 420 points should be considered as an experimental one, which tends to clarify the possibilities of applying the described method to the analysis of values of the T modulus. It does not seem possible to recommend, at this point, the coefficients obtained (Table 2) even for the entire surface surveyed by "Cosmos-49", to say nothing of the Earth itself.

In order to fully utilize the T modulus analysis it is necessary to consider the problems of factors affecting the coefficients obtained: the length of the harmonic progression; the number of points which have been measured and their distribution within the limits of the Earth surface; the representativeness of coefficients for points located outside the frame of measurements subject to subsequent analysis.

Table 2

	0	1	2	3	4	5	6
1	−30772	−1756					
2	− 1203	+2833	+1711				
3	+ 892	+1929	+1266	+ 853			
4	+ 793	+ 669	+ 696	− 489	+336		
5	− 380	+ 410	+ 242	− 35	−171	− 85	
6	+ 84	+ 139	− 20	− 260	+ 41	− 29	−72
1		+ 5978					
2		− 2415	+572				
3		− 234	+140	+190			
4		− 5	−126	−138	+ 40		
5		+ 85	+ 68	−166	− 86	+176	
6		− 52	+191	− 9	− 32	− 63	+162

In addition, the precision of the coefficients obtained could be affected by insufficient consideration of the Earth ellipticity at the time of the introduction of coordinates into the analysis. These problems will be further decided while studying experimental material obtained during surveys performed aboard the satellites, aeromagnetic and sea surveys and according to the observations of magnetic observatories as well as of maps with a good level of clarity.

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